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## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K-14 system for all students, especially those who have historically been underserved.
We work with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace-and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.
Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—Statistical Reasoning, Quantitative Reasoning, and Reasoning with Functions I and Reasoning with Functions II (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point-a developmental math course that helps students develop foundational skills and conceptual understanding in the context of collegelevel course material.
In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies-and tenacity-necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.
In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

## Acknowledgments

The development of the Dana Center Mathematics Pathways curricular materials began with the formation of the DCMP Curricular Design Team, who set the design standards for the curricular materials of individual DCMP courses. The team members are:

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The Usability Advisory Group members are:

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| 25.B | 25.B | Decaying Oscillations <br> Determine a formula for an exponential function to match given data | 409 | 329 | 502 | 25.B |
| 25.C | 25.C | Decaying Oscillations (Continued) Find a formula for a vertically shifted exponential function that matches given data | 412 | 331 | 509 | 25.C |
| 25.D | 25.D | Charging and Discharging Capacitors Determine the formula for a function composition | 416 | 333 | 514 | 25,.D |
| Lesson 26: Inverting Exponential Functions |  |  |  |  |  |  |
| 26.A | 26.A | Inverse Exponentials <br> Estimate input and output values for the inverse of exponential functions Sketch a graph of an inverse to an exponential function | 419 | 337 | 518 | 26.A |
| 26.B | 26.B | Logarithms <br> Compute the output of logarithm functions | 423 | 339 | 523 | 26.B |
| 26.C | 26.C | Graphing Logs Graph a logarithm function by hand | 427 | 343 | 530 | 26.C |
| 26.D | 26.D | Log Laws <br> Use laws of logarithms to expand a single logarithm into a sum or difference of logarithms | 431 | 345 | 535 | 26.D |
| 26.E | 26.E | Logarithmic Scales <br> Compare inputs and outputs of logarithmic functions <br> Use the laws of logarithms to simplify expressions | 436 | 349 | 541 | 26.E |


| $\begin{aligned} & \text { Ö } \\ & \text { U0 } \\ & 0 \end{aligned}$ |  | Lesson Title and Description |  |  |  |  |
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| Lesson 27: Solving Exponential and Logarithmic Equations |  |  |  |  |  |  |
| 27.A | 27.A | Savings Bonds <br> Solve exponential equations <br> Use the compound interest formula to calculate the doubling time for an investment Estimate the doubling time using the "rile of 72" | 441 | 353 | 547 | 27.A |
| 27.B | 27.B | How Do You Rank? <br> Solve exponential equations arising from logistic models and interpret the results | 446 | 357 | 553 | 27.B |
| 27.C | 27.C | Earthquake! <br> Solve equations containing one logarithmic expression | 449 | 359 | 559 | 27.C |
| 27.D | 27.D | Extraneous Solutions <br> Solve equations containing more than one logarithmic expression | 453 | 363 | 565 | 27.D |


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[^0]:    We welcome your comments and suggestions for improvements. Please contact us at danaweb@austin.utexas.edu or at the mailing address above.

