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# About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

# About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

- 1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
- 2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
- 3. Intentional use of strategies to help students develop skills as learners
- 4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning, Quantitative Reasoning,* and *Reasoning with Functions I* and *Reasoning with Functions II* (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

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# Acknowledgments

The development of the Dana Center Mathematics Pathways curricular materials began with the formation of the **DCMP Curricular Design Team**, who set the design standards for how the curricular materials for individual DCMP courses would be designed. The team members are:

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

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-	-	Curriculum Overview	xvi	-	-	-
-	-	Prep Week Ideas for your syllabus	xlvi	-	-	-
	Lesson 1	: Area Under a Curve and an Introduc	tion to Op	timizatio	n	
1.A	-	Approximating Area Approximate the area under the graph of a function using rectangles	1	1	1	1.A
1.B	1.B	Optimization and Rectangles Use technology to optimize the area and perimeter of rectangular regions	6	5	5	1.B
1.C	-	Our Learning Community Seek and give help	11	9	9	-
	Lesson 2	: Geometry of Triangles and the Pytha	gorean T	heorem		
2.A	2.A	Geometry of Triangles: Area Determine how much to reduce the height of a triangle in order to create a new triangle with a desired area	12	11	18	2.A
2.B	2.B	Right Triangles and Rates Determine the distance between two points in a plane using the Pythagorean theorem Determine the relationships between the rates of change among the sides of a dynamically changing right triangle	15	13	23	2.B
2.C	2.C	Distance and Arc Length Use the Pythagorean theorem to evaluate the distance between two points Use the Pythagorean theorem to estimate the length of a curve Use distances measured to determine average velocities	18	15	27	2.C

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	Lesson Preview Assignment	Lesson Title and Description : Rates of Change: Expanding Circles a	Purce Activities with Answers	B In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
		Circles and Rates of Change	<b>F</b> -			
		Determine the average rate of change of the circumference of a circle as a function of the average rate of change of the radius				
3.A	3.A	Determine the average rate of change of the area of a circle as a function of the average rate of change of the radius	22	19	31	3.A
		Determine the average rate of change of the volume of a disk as a function of the average rate of change of the radius				
		Spheres and Rates of Change				
3.B	3.B	Use the relationship between volume and radius of a sphere to determine the relationship between their rates of change	25	21	36	3.B
		Compute rates of change				
3.C	-	Forming Effective Study Groups Describe how to form and conduct an effective study group Identify key characteristics of effective study groups	29	23	41	-
	Lesson 4	: Equations of Circles				
4.A	4.A	Distance on the Line Compute the distance between two numbers on the number line as represented by an absolute value Solve equations and inequalities involving absolute values Interpret intervals using absolute value notation	31	25	45	4.A

	Lesson Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
		Circles Given a point, P and a positive distance, r,				
		determine an equation whose graph is a circle centered at P and with radius r				
4.B	4.B	<i>Given two points in the plane, determine the equation of the smallest circle containing both points</i>	36	29	50	4.B
		Given two points in the plane, determine the center and radius of the smallest circle containing both points				
		Equations of Circles				
4.C	4.C	Determine the center and radius of the graph of a quadratic equation when the graph is a circle	39	31	54	4.C
		Sketch the graph of a quadratic equation				
	Lesson 5	: Similar Triangles, Circular Motion, a	nd Measu	ring Angl	es	
		Moving Shadows				
5.A	5.A	Use similar triangles to model static relationships between quantities Use similar triangles to model dynamic relationships between quantities	43	33	60	5.A
		Home Improvement				
5.B	5.B	Set up and solve geometric optimization problems	47	37	65	5.B
		Maximize the area of a rectangle inscribed inside an equilateral triangle				
		You Spin Me Round				
		Calculate the speed of an object in uniform circular motion				
5.C	5.C	Use proportional reasoning to find arc lengths and areas of circular sectors	52	39	71	5.C
		Determine the radian measure of the central angle of a given circular sector				

	Lesson Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
	Lesson 6	: Related Rates and Optimization: Con	les and Cy	linders		
6.A	6.A	Red Plastic Cup Calculate the surface area and volume of a cylinder Calculate surface areas and volumes of cones and frustrums	56	43	77	6.A
6.B	6.B	Can It! Determine the dimensions of a cylinder of given volume with minimum surface area	60	47	82	6.B
6.C	6.C	Off to a Rocky Start Model the changing dimensions in a cone Compute the average rates of change of dimensions in a cone Estimate instantaneous rates of change of dimensions in a cone	64	51	87	6.C
6.D	6.D	A Geometric Problem Determine the dimensions of a cylinder inscribed in a cone which produces the greatest volume	69	55	92	6.D
	Lesson 7	': Sinusoidal Models				
7.A	7.A	Modeling Tides Sketch a graph of a function based on data to model a physical situation Interpret a mathematical model of a physical situation and use the model to make decisions	72	57	96	7.A
7.B	7.B	Pendulum Motion Identify the period of a sinusoidal function from its graph Given the graph of a sinusoidal position function, sketch the graph of the corresponding velocity function	77	61	101	7.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
7.C	7.C	Modeling Temperature Change Identify the period, amplitude, and midline of a sinusoidal function given its graph Interpret the meanings of period, amplitude, and midline in the context of a model Create a graphical approximation of a sinusoidal function which models given data	82	65	105	7.C
I	Lesson 8	: The Unit Circle				
8.A	8.A	Constructing Sinusoidals From Circles Graph the horizontal and vertical coordinates of a point as it moves around a circle Interpret circle diagrams in the context of a model	87	69	109	8.A
8.B	8.B	The Sine and Cosine Functions Compute some important values of the sine and cosine functions using the unit circle Evaluate and graph the sine and cosine functions using a graphing calculator or app, using radians appropriately Interpret formulas for sinusoidal functions in the context of a model	91	73	112	8.B
8.C	8.C	Special Angles Locate special angles on the unit circle Give measures of special angles on the unit circle, both in degrees and in radians Compute exact values for the sine and cosine of these special angles	95	77	116	8.C
8.D	8.D	Special Values of Sinusoidal Functions Locate the special points on the graphs of sinusoidal functions Interpret the special points of a sinusoidal function in the context of a model	101	81	120	8.D

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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
	Lesson 9	: Circles and Sinusoidal Models				
		Non-Unit Circles <i>Model a physical situation using a non- unit circle</i>				
9.A	9.A	Sketch the graph of a sine or cosine function represented by a non-unit circle Fund a formula for a sine or cosine function represented by a non unit circle	105	85	124	9.A
9.B	9.B	Changes in Angle and Radius Find the (x,y) coordinates of points on a nonunit circle Find the radius of a non-unit circle and an angle to correspond to a point (x,y) Describe how changes in angle and radius affect the location of points	110	89	130	9.B
9.C	9.C	Damped Harmonic Motion Model damped harmonic motion with a function using formulas and graphs	114	93	135	9.C
	Lesson 1	0: Analyzing Sinusoidal Functions				
10.A	10.A	Modeling the Motion of a Pendulum Write and graph equations that model the oscillations of a pendulum Identify those parameters that affect the amplitude of a pendulum's motion	118	97	139	10.A
10.B	10.B	Modeling Cell Phone Signals Write equations that model the oscillations of a pendulum Identify those parameters that affect the amplitude of a pendulum's motion	123	101	143	10.B
10.C	10.C	Modeling the Vibration of a Cell Phone Determine the amount of horizontal shift present in a sine function	129	105	148	10.C

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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
	Lesson 1	1: Transformations of Sinusoidal Fun	ctions			
11.A	11.A	Staying Current Around the World Starting with a formula, calculate the period and amplitude of a sine function and use this information to produce a graph of the function Estimate the period and amplitude of a sine function from its graph	135	109	153	11.A
11.B	11.B	Periodic Models with Vertical Shifts Determine maximum and minimum values (and when they occur) of a sinusoidal model Determine the period of a sinusoidal model Explain the similarities and differences after a function has undergone a vertical shift	141	113	158	11.B
11.C	11.C	Periodic Models with Horizontal Shifts Make appropriate changes to an algebraic model to result in the necessary horizontal shift Discuss how different parameters will affect the amplitude, period, vertical shift, and horizontal shifts of sine functions	144	115	162	11.C
	Lesson 1	2: Describing Change in Sinusoidal Fu	nctions			
12.A	12.A	Rate of Change of Sine Compute the average rate of change of sin x Determine a formula for the average rate of change of sin x Use technology to graph the average rate of change of sin x	148	119	166	12.A
12.B	12.B	A Closer Look at Rate of Change of Sine Examine, compute, and compare the maximum average rate of change for a variety of sine functions	151	121	170	12.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
12.C	12.C	Applications of Rate of Change of Sine Use a graph to estimate where the greatest average rate of change may occur Calculate the average rate of change of a complex function Determine when oscillations have decreased below a given criterion	162	125	175	12.C
12.D	12.D	Amplitude Decay of Sine Functions Determine the formula for a function given its graph Identify how the amplitude of a given function decays	169	129	179	12.D
I	Lesson 1	3: Right Triangle Trigonometry				
13.A	13.A	From Circles to Triangles Use sine and cosine to determine side lengths of a right triangle	172	131	183	13.A
13.B	13.B	From Circles to Triangles (Continued) Use sine and cosine to determine side lengths of a right triangle	177	135	187	13.B
13.C	13.C	Hypotenuse Trouble Use the right triangle definitions of sine and cosine to find the hypotenuse of a right triangle when given a leg and an acute angle Fund a second leg using the Pythagorean theorem once the hypotenuse and the first leg are known	180	137	192	13.C
13.D	13.D	A Sine of Things to Come Solve for a missing leg of a right triangle when given one leg and an acute angle without solving for the hypotenuse first	182	139	196	13.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
L	esson 1	4: Inverse Trigonometric Functions				
14.A	14.A	Does Inverse Cosine Exist? Identify a reasonable restricted domain for the cosine function Determine the domain and range of the inverse cosine function Evaluate inverse cosine at several special values	185	143	200	14.A
14.B	14.B	Understanding the Inverse Cosine Functions Plot points on the graph of an inverse function given points on the graph of the function Use the graph of a function to help determine the steepness of the graph of the inverse function Sketch a graph of the inverse cosine function	189	147	204	14.B
14.C	14.C	Is This Ladder Safe? Use inverse sine and cosine to determine when a ladder is being used safely Solve simple expressions using inverse sine and cosine	193	151	208	14.C
L	esson 1	5: Solving Trigonometric Equations				
15.A	15.A	Equations Involving Sine and Cosine Solve for an unknown angle in an equation involving sine or cosine Use a calculator or app to evaluate inverse sine and inverse cosine Use a model involving inverse trigonometric functions to make decisions about a physical situation	196	153	212	15.A
15.B	15.B	Solving for Obtuse Angles Find angle measures in the first and second quadrants corresponding to values of sine and cosine Determine the correct angle corresponding to a given value of sine or cosine, in the context of a problem	200	157	217	15.B

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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
15.C	15.C	Choosing the Quadrant Solve and Equation involving sine or cosine for an unknown angle in a specified quadrant or interval Find multiple solutions for a trigonometric equation	204	161	222	15.C
15.D	15.D	Solving Trigonometric Equations Find all of the solutions to an equation involving sine or cosine within a specified interval	209	165	226	15.D
Le	esson 1	6: The Pythagorean Identity and Pola	r Curves			
16.A	16.A	The Pythagorean Identity Prove identities using the Pythagorean identity Use the Pythagorean identity to find values of sine and cosine	213	169	231	16.A
16.B	16.B	Is My Answer Right? Use the Pythagorean identity to rewrite trigonometric expressions in equivalent forms	216	171	236	16.B
16.C	16.C	Polar Graphs Plot Points in Polar Coordinates Use Technology to produce graphs of polar curves	220	173	241	16.C
16.D	16.D	A Gallery of Polar Curves Convert a Cartesian equation to polar form Graph polar curves using technology	225	117	247	16.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
L	esson 1	7: Sum and Difference Identities				
17.A	17.A	Angle and Sum Identities <i>Apply the angle sum identities for sine</i> <i>and cosine</i>	232	181	252	17.A
17.B	17.B	What's the Difference Manipulate trigonometric expressions using the angle sum and difference identities for sine and cosine	236	183	256	17.B
17.C	17.C	Guitar Harmonics Manipulate trigonometric expressions using the angle sum and difference formulas for sine and cosine Determine the locations of the nodes of a standing wave	241	185	260	17.C
17.D	17.D	In Tune Use a sum-to-product identity to rewrite and analyze the sum of two sine functions	246	189	266	17.D
		Lesson 18: Double and Half-A	ngle Form	ulas		
18.A	18.A	Projectile Motion Use the double-angle formula for sine to maximize certain trigonometric expressions Use the double-angle formula to determine the sine of twice an angle based on the sine and cosine of the original angle	250	191	272	18.A
18.B	18.B	Malus' Law Use the double angle formula for cosine to solve equations involving a cos <sup>2</sup> (x) Use the double angle formula to determine the cosine of twice an angle based on the sine and cosine of the original angle	255	195	277	18.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
18.C	18.C	Planetary Motion Use the half angle formulas to compute the sine and cosine values of a half angle based on the sine and cosine values of the original angle	259	199	281	18.C
18.D	18.D	Circular Motion Solve a trigonometric equation by obtaining common arguments for all trigonometric functions	264	203	286	18.D
L	esson 1	9: Law of Sines and Law of Cosines				
19.A	19.A	The Montreal Tower Solve oblique triangles in which two angles and one side are known	268	205	290	19.A
19.B	19.B	Can You Hear Me Now? Use the Law of Sines to determine missing angles in triangles	272	209	295	19.B
19.C	19.C	Play Ball! Use the Law od Cosines to find the missing side of a triangle when two sides and the angle between them are given	277	213	300	19.C
19.D	19.D	Here Comes the Sun Find the missing angles in a triangle when all three sides are known	282	217	304	19.D
19.E	19.E	Sines or Cosines? Develop a strategy for solving a given oblique triangle	286	219	309	19.E
L	esson 2	0: Secant and Tangent Functions				
20.A	20.A	The Tangent Function Use the tangent function to determine unknown lengths in a right triangle	290	221	315	20.A
20.B	20.B	Graphing the Tangent Function Sketch the graph of the tangent function Interpret the graph of the tangent function in the context of a model	295	225	319	20.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
20.C	20.C	The Inverse Tangent Function Use the inverse tangent function to model a physical situation Use the inverse tangent function to find an unknown angle in a right triangle	300	229	323	20.C
20.D	20.D	The Secant Function Use the decant function to compute missing lengths of a right triangle	304	231	327	20.D
20.E	20.E	Identities Involving Secant and Tangent Derive identities for tangent and secant using the identities for sine and cosine Interpret the identities for tangent and secant in the context of a physical problem	308	235	331	20.E
L	esson 2	1: Cosecant and Cotangent Functions				
21.A	21.A	The Cotangent Function Use the cotangent function to solve problems involving right triangles Interpret the graph of the cotangent function in the context of a model	312	239	335	21.A
21.B	21.B	Inverting the Cotangent Function Solve an equation involving the cotangent function for an unknown angle Graph the inverse cotangent function using a calculator or app and interpret the graph in the context of a model	317	243	339	21.B
21.C	21.C	The Cosecant Function Model relationships between quantities using the cosecant function Interpret the graph of the cosecant function in the context of a model Derive some identities involving the cosecant funtion	321	245	345	21.C

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
		Inverses of Secant and Cosecant Write the inverse secant and the inverse cosecant in terms of the inverse sine and the inverse cosine				
21.D	21.D	Find the inverses of functions involving secant and cosecant Interpret inverse function involving	325	249	350	21.D
		secant and cosecant in the context of a model				
L	esson 2	2: Applications of Periodic Functions				
22.A	22.A	Rising Carbon Dioxide Develop a model that incorporates both a large-scale trend along with a smaller scale cyclic behavior	329	253	354	22.A
22.B	22.B	Car Wheels Develop a model that incorporates both linear and cyclic behavior	332	255	358	22.B
22.C	22.C	Train Wheels Develop a model that incorporated both linear and cyclic behavior	336	257	363	22.C
22.D	22.D	Amplitude Modulation Explain how AM radio signals encode information Extract the message and carrier signals from the graph of an AM radio signal	339	259	368	22.D

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