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## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K-14 system for all students, especially those who have historically been underserved.
We work with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace-and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.
Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.
We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.
We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways-Statistical Reasoning, Quantitative Reasoning, and Reasoning with Functions I and Reasoning with Functions II (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point-a developmental math course that helps students develop foundational skills and conceptual understanding in the context of collegelevel course material.
In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies-and tenacity-necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.
In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

## Acknowledgments

The development of the Dana Center Mathematics Pathways curricular materials began with the formation of the DCMP Curricular Design Team, who set the design standards for how the curricular materials for individual DCMP courses would be designed. The team members are:

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The Usability Advisory Group members are:
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## Contents

| $\begin{aligned} & 0 \\ & 0 \\ & \text { ® } \\ & \hline \end{aligned}$ |  | Lesson Title and Description |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | - | Curriculum Overview | xvi | - | - | - |
| - | - | Prep Week <br> Ideas for your syllabus | xlvi | - | - | - |
| Lesson 1: Area Under a Curve and an Introduction to Optimization |  |  |  |  |  |  |
| 1.A | - | Approximating Area <br> Approximate the area under the graph of a function using rectangles | 1 | 1 | 1 | 1.A |
| 1.B | 1.B | Optimization and Rectangles <br> Use technology to optimize the area and perimeter of rectangular regions | 6 | 5 | 5 | 1.B |
| 1.C | - | Our Learning Community Seek and give help | 11 | 9 | 9 |  |

Lesson 2: Geometry of Triangles and the Pythagorean Theorem

| 2.A | 2.A | Geometry of Triangles: Area |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Determine how much to reduce the height of a triangle in order to create a new triangle with a desired area | 12 | 11 | 18 | 2.A |
| 2.B | 2.B | Right Triangles and Rates | 15 | 13 | 23 | 2.B |
|  |  | Determine the distance between two points in a plane using the Pythagorean theorem |  |  |  |  |
|  |  | Determine the relationships between the rates of change among the sides of a dynamically changing right triangle |  |  |  |  |
| 2.C | 2.C | Distance and Arc Length | 18 | 15 | 27 | 2.C |
|  |  | Use the Pythagorean theorem to evaluate the distance between two points |  |  |  |  |
|  |  | Use the Pythagorean theorem to estimate the length of a curve |  |  |  |  |
|  |  | Use distances measured to determine average velocities |  |  |  |  |


|  |  | Lesson Title and Description | 志 |  |  | Practice Assignment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 3: Rates of Change: Expanding Circles and Spheres |  |  |  |  |  |  |
| 3.A | 3.A | Circles and Rates of Change <br> Determine the average rate of change of the circumference of a circle as a function of the average rate of change of the radius <br> Determine the average rate of change of the area of a circle as a function of the average rate of change of the radius <br> Determine the average rate of change of the volume of a disk as a function of the average rate of change of the radius | 22 | 19 | 31 | 3.A |
| 3.B | $3 . \mathrm{B}$ | Spheres and Rates of Change <br> Use the relationship between volume and radius of a sphere to determine the relationship between their rates of change <br> Compute rates of change | 25 | 21 | 36 | 3.B |
| 3.C | - | Forming Effective Study Groups <br> Describe how to form and conduct an effective study group <br> Identify key characteristics of effective study groups | 29 | 23 | 41 | - |
| Lesson 4: Equations of Circles |  |  |  |  |  |  |
| 4.A | $4 . \mathrm{A}$ | Distance on the Line <br> Compute the distance between two numbers on the number line as represented by an absolute value <br> Solve equations and inequalities involving absolute values <br> Interpret intervals using absolute value notation | 31 | 25 | 45 | 4.A |


| İ Oid O |  | Lesson Title and Description |  | $\begin{aligned} & \text { In-Class Activities } \\ & \text { (Student) } \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.B | 4.B | Circles | 36 | 29 | 50 | 4.B |
|  |  | Given a point, P and a positive distance, r , determine an equation whose graph is a circle centered at P and with radius r |  |  |  |  |
|  |  | Given two points in the plane, determine the equation of the smallest circle containing both points |  |  |  |  |
|  |  | Given two points in the plane, determine the center and radius of the smallest circle containing both points |  |  |  |  |
| 4.C | 4.C | Equations of Circles | 39 | 31 | 54 | 4.C |
|  |  | Determine the center and radius of the graph of a quadratic equation when the graph is a circle |  |  |  |  |
|  |  | Sketch the graph of a quadratic equation |  |  |  |  |
| Lesson 5: Similar Triangles, Circular Motion, and Measuring Angles |  |  |  |  |  |  |
| 5.A | 5.A | Moving Shadows | 43 | 33 | 60 | 5.A |
|  |  | Use similar triangles to model static relationships between quantities |  |  |  |  |
|  |  | Use similar triangles to model dynamic relationships between quantities |  |  |  |  |
| 5.B | 5.B | Home Improvement | 47 | 37 | 65 | 5.B |
|  |  | Set up and solve geometric optimization problems |  |  |  |  |
|  |  | Maximize the area of a rectangle inscribed inside an equilateral triangle |  |  |  |  |
| 5.C | 5.C | You Spin Me Round | 52 | 39 | 71 | 5.C |
|  |  | Calculate the speed of an object in uniform circular motion |  |  |  |  |
|  |  | Use proportional reasoning to find arc lengths and areas of circular sectors |  |  |  |  |
|  |  | Determine the radian measure of the central angle of a given circular sector |  |  |  |  |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 6: Related Rates and Optimization: Cones and Cylinders |  |  |  |  |  |  |
| 6.A | 6.A | Red Plastic Cup <br> Calculate the surface area and volume of a cylinder <br> Calculate surface areas and volumes of cones and frustrums | 56 | 43 | 77 | 6.A |
| 6.B | 6.B | Can It! <br> Determine the dimensions of a cylinder of given volume with minimum surface area | 60 | 47 | 82 | 6.B |
| 6.C | 6.C | Off to a Rocky Start <br> Model the changing dimensions in a cone <br> Compute the average rates of change of dimensions in a cone <br> Estimate instantaneous rates of change of dimensions in a cone | 64 | 51 | 87 | 6.C |
| 6.D | 6.D | A Geometric Problem <br> Determine the dimensions of a cylinder inscribed in a cone which produces the greatest volume | 69 | 55 | 92 | 6.D |
| Lesson 7: Sinusoidal Models |  |  |  |  |  |  |
| 7.A | 7.A | Modeling Tides <br> Sketch a graph of a function based on data to model a physical situation <br> Interpret a mathematical model of a physical situation and use the model to make decisions | 72 | 57 | 96 | 7.A |
| 7.B | 7.B | Pendulum Motion <br> Identify the period of a sinusoidal function from its graph <br> Given the graph of a sinusoidal position function, sketch the graph of the corresponding velocity function | 77 | 61 | 101 | 7.B |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.C | 7.C | Modeling Temperature Change <br> Identify the period, amplitude, and midline of a sinusoidal function given its graph <br> Interpret the meanings of period, amplitude, and midline in the context of a model <br> Create a graphical approximation of a sinusoidal function which models given data | 82 | 65 | 105 | 7.C |
| Lesson 8: The Unit Circle |  |  |  |  |  |  |
| 8.A | 8.A | Constructing Sinusoidals From Circles <br> Graph the horizontal and vertical coordinates of a point as it moves around a circle <br> Interpret circle diagrams in the context of a model | 87 | 69 | 109 | 8.A |
| 8.B | 8.B | The Sine and Cosine Functions <br> Compute some important values of the sine and cosine functions using the unit circle <br> Evaluate and graph the sine and cosine functions using a graphing calculator or app, using radians appropriately <br> Interpret formulas for sinusoidal functions in the context of a model | 91 | 73 | 112 | 8.B |
| 8.C | 8.C | Special Angles <br> Locate special angles on the unit circle Give measures of special angles on the unit circle, both in degrees and in radians <br> Compute exact values for the sine and cosine of these special angles | 95 | 77 | 116 | 8.C |
| 8.D | 8.D | Special Values of Sinusoidal Functions <br> Locate the special points on the graphs of sinusoidal functions <br> Interpret the special points of a sinusoidal function in the context of a model | 101 | 81 | 120 | 8.D |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 9: Circles and Sinusoidal Models |  |  |  |  |  |  |
| 9.A | 9.A | Non-Unit Circles <br> Model a physical situation using a nonunit circle <br> Sketch the graph of a sine or cosine function represented by a non-unit circle <br> Fund a formula for a sine or cosine function represented by a non unit circle | 105 | 85 | 124 | $9 . \mathrm{A}$ |
| 9.B | 9.B | Changes in Angle and Radius <br> Find the ( $\mathrm{x}, \mathrm{y}$ ) coordinates of points on a nonunit circle <br> Find the radius of a non-unit circle and an angle to correspond to a point ( $\mathrm{x}, \mathrm{y}$ ) <br> Describe how changes in angle and radius affect the location of points | 110 | 89 | 130 | 9.B |
| 9.C | 9.C | Damped Harmonic Motion <br> Model damped harmonic motion with a function using formulas and graphs | 114 | 93 | 135 | $9 . \mathrm{C}$ |
| Lesson 10: Analyzing Sinusoidal Functions |  |  |  |  |  |  |
| 10.A | 10.A | Modeling the Motion of a Pendulum <br> Write and graph equations that model the oscillations of a pendulum <br> Identify those parameters that affect the amplitude of a pendulum's motion | 118 | 97 | 139 | 10.A |
| 10.B | 10.B | Modeling Cell Phone Signals <br> Write equations that model the oscillations of a pendulum <br> Identify those parameters that affect the amplitude of a pendulum's motion | 123 | 101 | 143 | 10.B |
| 10.C | 10.C | Modeling the Vibration of a Cell Phone Determine the amount of horizontal shift present in a sine function | 129 | 105 | 148 | 10.C |


|  |  | Lesson Title and Description | In-Class Activities with Answers |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 11: Transformations of Sinusoidal Functions |  |  |  |  |  |  |
| 11.A | 11.A | Staying Current Around the World <br> Starting with a formula, calculate the period and amplitude of a sine function and use this information to produce a graph of the function <br> Estimate the period and amplitude of a sine function from its graph | 135 | 109 | 153 | 11.A |
| 11.B | 11.B | Periodic Models with Vertical Shifts <br> Determine maximum and minimum values (and when they occur) of a sinusoidal model <br> Determine the period of a sinusoidal model <br> Explain the similarities and differences after a function has undergone a vertical shift | 141 | 113 | 158 | 11.B |
| 11.C | $11 . \mathrm{C}$ | Periodic Models with Horizontal Shifts <br> Make appropriate changes to an algebraic model to result in the necessary horizontal shift <br> Discuss how different parameters will affect the amplitude, period, vertical shift, and horizontal shifts of sine functions | 144 | 115 | 162 | 11.C |
| Lesson 12: Describing Change in Sinusoidal Functions |  |  |  |  |  |  |
| 12.A | 12.A | Rate of Change of Sine <br> Compute the average rate of change of $\sin \mathrm{x}$ <br> Determine a formula for the average rate of change of $\sin \mathrm{x}$ <br> Use technology to graph the average rate of change of $\sin \mathrm{x}$ | 148 | 119 | 166 | 12.A |
| 12.B | 12.B | A Closer Look at Rate of Change of Sine Examine, compute, and compare the maximum average rate of change for a variety of sine functions | 151 | 121 | 170 | 12.B |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12.C | 12.C | Applications of Rate of Change of Sine Use a graph to estimate where the greatest average rate of change may occur <br> Calculate the average rate of change of a complex function <br> Determine when oscillations have decreased below a given criterion | 162 | 125 | 175 | 12.C |
| 12.D | 12.D | Amplitude Decay of Sine Functions Determine the formula for a function given its graph <br> Identify how the amplitude of a given function decays | 169 | 129 | 179 | 12.D |
| Lesson 13: Right Triangle Trigonometry |  |  |  |  |  |  |
| 13.A | 13.A | From Circles to Triangles <br> Use sine and cosine to determine side lengths of a right triangle | 172 | 131 | 183 | 13.A |
| 13.B | 13.B | From Circles to Triangles (Continued) <br> Use sine and cosine to determine side lengths of a right triangle | 177 | 135 | 187 | 13.B |
| 13.C | 13.C | Hypotenuse Trouble <br> Use the right triangle definitions of sine and cosine to find the hypotenuse of a right triangle when given a leg and an acute angle <br> Fund a second leg using the Pythagorean theorem once the hypotenuse and the first leg are known | 180 | 137 | 192 | 13.C |
| 13.D | 13.D | A Sine of Things to Come Solve for a missing leg of a right triangle when given one leg and an acute angle without solving for the hypotenuse first | 182 | 139 | 196 | 13.D |


|  | Preview Assignment | Lesson Title and Description |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 14: Inverse Trigonometric Functions |  |  |  |  |  |  |
| 14.A | 14.A | Does Inverse Cosine Exist? <br> Identify a reasonable restricted domain for the cosine function <br> Determine the domain and range of the inverse cosine function <br> Evaluate inverse cosine at several special values | 185 | 143 | 200 | 14.A |
| 14.B | 14.B | Understanding the Inverse Cosine <br> Functions <br> Plot points on the graph of an inverse function given points on the graph of the function <br> Use the graph of a function to help determine the steepness of the graph of the inverse function <br> Sketch a graph of the inverse cosine function | 189 | 147 | 204 | 14.B |
| 14.C | 14.C | Is This Ladder Safe? <br> Use inverse sine and cosine to determine when a ladder is being used safely <br> Solve simple expressions using inverse sine and cosine | 193 | 151 | 208 | 14.C |
| Lesson 15: Solving Trigonometric Equations |  |  |  |  |  |  |
| 15.A | 15.A | Equations Involving Sine and Cosine Solve for an unknown angle in an equation involving sine or cosine Use a calculator or app to evaluate inverse sine and inverse cosine Use a model involving inverse trigonometric functions to make decisions about a physical situation | 196 | 153 | 212 | 15.A |
| 15.B | 15.B | Solving for Obtuse Angles <br> Find angle measures in the first and second quadrants corresponding to values of sine and cosine <br> Determine the correct angle corresponding to a given value of sine or cosine, in the context of a problem | 200 | 157 | 217 | 15.B |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15.C | 15.C | Choosing the Quadrant <br> Solve and Equation involving sine or cosine for an unknown angle in a specified quadrant or interval <br> Find multiple solutions for a trigonometric equation | 204 | 161 | 222 | 15.C |
| 15.D | 15.D | Solving Trigonometric Equations <br> Find all of the solutions to an equation involving sine or cosine within a specified interval | 209 | 165 | 226 | 15.D |
| Lesson 16: The Pythagorean Identity and Polar Curves |  |  |  |  |  |  |
| 16.A | 16.A | The Pythagorean Identity <br> Prove identities using the Pythagorean identity <br> Use the Pythagorean identity to find values of sine and cosine | 213 | 169 | 231 | 16.A |
| 16.B | 16.B | Is My Answer Right? <br> Use the Pythagorean identity to rewrite trigonometric expressions in equivalent forms | 216 | 171 | 236 | 16.B |
| 16.C | 16.C | Polar Graphs <br> Plot Points in Polar Coordinates <br> Use Technology to produce graphs of polar curves | 220 | 173 | 241 | 16.C |
| 16.D | 16.D | A Gallery of Polar Curves <br> Convert a Cartesian equation to polar form <br> Graph polar curves using technology | 225 | 117 | 247 | 16.D |


| $\begin{aligned} & \text { E } \\ & \text { Un } \\ & \text { O} \end{aligned}$ | Preview Assignment | Lesson Title and Description |  | $\begin{aligned} & \text { In-Class Activities } \\ & \text { (Student) } \end{aligned}$ |  | Practice Assignment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 17: Sum and Difference Identities |  |  |  |  |  |  |
| 17.A | 17.A | Angle and Sum Identities <br> Apply the angle sum identities for sine and cosine | 232 | 181 | 252 | 17.A |
| 17.B | 17.B | What's the Difference <br> Manipulate trigonometric expressions using the angle sum and difference identities for sine and cosine | 236 | 183 | 256 | 17.B |
| 17.C | 17.C | Guitar Harmonics <br> Manipulate trigonometric expressions using the angle sum and difference formulas for sine and cosine <br> Determine the locations of the nodes of a standing wave | 241 | 185 | 260 | 17.C |
| 17.D | 17.D | In Tune <br> Use a sum-to-product identity to rewrite and analyze the sum of two sine functions | 246 | 189 | 266 | 17.D |
| Lesson 18: Double and Half-Angle Formulas |  |  |  |  |  |  |
| 18.A | 18.A | Projectile Motion <br> Use the double-angle formula for sine to maximize certain trigonometric expressions <br> Use the double-angle formula to determine the sine of twice an angle based on the sine and cosine of the original angle | 250 | 191 | 272 | 18.A |
| 18.B | 18.B | Malus' Law <br> Use the double angle formula for cosine to solve equations involving a $\cos ^{2}(x)$ <br> Use the double angle formula to determine the cosine of twice an angle based on the sine and cosine of the original angle | 255 | 195 | 277 | 18.B |


| $\begin{aligned} & \text { E } \\ & \text { Di } \\ & 0 \end{aligned}$ | Preview Assignment | Lesson Title and Description |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18.C | 18.C | Planetary Motion <br> Use the half angle formulas to compute the sine and cosine values of a half angle based on the sine and cosine values of the original angle | 259 | 199 | 281 | 18.C |
| 18.D | 18.D | Circular Motion <br> Solve a trigonometric equation by obtaining common arguments for all trigonometric functions | 264 | 203 | 286 | 18.D |
| Lesson 19: Law of Sines and Law of Cosines |  |  |  |  |  |  |
| 19.A | 19.A | The Montreal Tower <br> Solve oblique triangles in which two angles and one side are known | 268 | 205 | 290 | 19.A |
| 19.B | 19.B | Can You Hear Me Now? <br> Use the Law of Sines to determine missing angles in triangles | 272 | 209 | 295 | 19.B |
| 19.C | 19.C | Play Ball! <br> Use the Law od Cosines to find the missing side of a triangle when two sides and the angle between them are given | 277 | 213 | 300 | 19.C |
| 19.D | 19.D | Here Comes the Sun <br> Find the missing angles in a triangle when all three sides are known | 282 | 217 | 304 | 19.D |
| 19.E | 19.E | Sines or Cosines? <br> Develop a strategy for solving a given oblique triangle | 286 | 219 | 309 | 19.E |
| Lesson 20: Secant and Tangent Functions |  |  |  |  |  |  |
| 20.A | 20.A | The Tangent Function <br> Use the tangent function to determine unknown lengths in a right triangle | 290 | 221 | 315 | 20.A |
| 20.B | 20.B | Graphing the Tangent Function Sketch the graph of the tangent function Interpret the graph of the tangent function in the context of a model | 295 | 225 | 319 | 20.B |


|  | 301 |
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| Overview | 1 |
| :---: | :---: |
| Angle Measure | 3 |
| Arithmetic with Fractions | 6 |
| Combining Like Terms | 14 |
| Coordinate Plane | 17 |
| Dimensional Analysis | 21 |
| Distributive Property | 27 |
| Exponent Rules | 29 |
| Factoring | 32 |
| Factoring Polynomials | 41 |
| Four Representations of Functions | 47 |
| Geometry | 50 |
| Graphing Technology | 60 |
| Lines | 76 |
| Order of Operations | 81 |
| Parabolas and Quadratic Functions | 82 |
| Roots and Radicals | 88 |
| Scientific Notation | 94 |
| Slope | 96 |
| Solving Quadratic Equations | 98 |
| Sums and Differences of Cubes | 103 |


| Transformations | 105 |
| :---: | :---: |
| Trigonometric Formulas | 110 |
| Writing Principles | 119 |
| Glossary | Glossary -1 |

